

St. Philip's Lutheran **Mathematics Content Standards**

Introduction and Rationale

The study of mathematics is important to help students recognize the orderly way God created our world and how in that same way he allows numbers to work perfectly together as calculations are performed. Knowledge of mathematics is essential to be a productive citizen. St. Philip's is committed to offering students a high quality mathematics education so they are prepared to be leaders in the church, school, and community.

The study of mathematics also provides every student with the opportunity to choose among the full range of future career paths. Mathematics trains the mind to be analytic - providing the foundation for intelligent and precise thinking.

To compete successfully in the worldwide economy, students must have a high degree of comprehension in mathematics. More than that, however, mathematics is critical for all students, not only those who will have careers that demand advanced mathematical preparation, but all citizens who will be living in the twenty-first century. These standards are based on the premise that all students are capable of learning rigorous mathematics and learning it well. Proficiency in most of mathematics is not an innate characteristic; it is achieved through persistence, effort, and practice.

The standards focus on essential content for all students and prepare students for the study of advanced mathematics, science and technical careers, and postsecondary study in all content areas. All students are required to grapple with solving problems; develop abstract, analytic thinking skills; learn to deal effectively and comfortably with variables and equations; and use mathematical notation effectively to model situations. The goal in mathematics education is for students to:

- Develop fluency in basic computational skills.

- Develop an understanding of mathematical concepts.

- Become mathematical problem solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent.

- Communicate precisely about quantities, logical relationships, and unknown values through the use of signs, symbols, models, graphs, and mathematical terms.

- Reason mathematically by gathering data, analyzing evidence, and building arguments to support or refute hypotheses.

- Make connections among mathematical ideas and between mathematics and other disciplines.

The standards identify what all students should know and be able to do at each grade level. The standards emphasize computational and procedural skills, conceptual understanding, and problem solving. These three components of mathematics instruction and learning are not separate from each other; instead, they are intertwined and mutually reinforcing.

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Mathematics Content Standards

Pre-Kindergarten

By the end of pre-kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way.

Number Sense

- 1. Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):**
 - a. Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other.
 - b. Count, recognize, represent, name, and order a number of objects (up to 30).
 - c. Know that the larger numbers describe sets with more objects in them than the smaller numbers have.
- 2. Students understand and describe simple additions and subtractions:**
 - a. Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).
- 3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:**
 - a. Recognize when an estimate is reasonable.

Algebra and Functions

- 4. Students sort and classify objects:**
 - a. Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).

Measurement and Geometry

- 5. Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:**
 - a. Compare the length, weight, and capacity of objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).
 - b. Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar).
 - c. Name the days of the week.
 - d. Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).
- 6. Students identify common objects in their environment and describe the geometric features:**
 - a. Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).
 - b. Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Statistics, Data Analysis, and Probability

7. Students collect information about objects and events in their environment:

- a. Pose information questions; collect data; and record the results using objects, pictures, and picture graphs.
- b. Identify, describe, and extend simple patterns (such as circles or triangles) by referring to their shapes, sizes, or colors.

Mathematical Reasoning

8. Students make decisions about how to set up a problem:

- a. Determine the approach, materials, and strategies to be used.
- b. Use tools and strategies, such as manipulatives or sketches, to model problems.

9. Students solve problems in reasonable ways and justify their reasoning:

- a. Explain the reasoning used with concrete objects and/ or pictorial representations.
- b. Make precise calculations and check the validity of the results in the context of the problem.

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Mathematics Content Standards

Kindergarten

By the end of kindergarten, students understand small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense of properties and patterns. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way.

Number Sense

- 1. Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement):**
 - a. Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other.
 - b. Count, recognize, represent, name, and order a number of objects (up to 30).
 - c. Know that the larger numbers describe sets with more objects in them than the smaller numbers have.
- 2. Students understand and describe simple additions and subtractions:**
 - a. Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).
- 3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places:**
 - a. Recognize when an estimate is reasonable.

Algebra and Functions

- 4. Students sort and classify objects:**
 - a. Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).

Measurement and Geometry

- 5. Students understand the concept of time and units to measure it; they understand that objects have properties, such as length, weight, and capacity, and that comparisons may be made by referring to those properties:**
 - a. Compare the length, weight, and capacity of objects by making direct comparisons with reference objects (e.g., note which object is shorter, longer, taller, lighter, heavier, or holds more).
 - b. Demonstrate an understanding of concepts of time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar).
 - c. Name the days of the week.
 - d. Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock; bedtime is 8 o'clock at night).
- 6. Students identify common objects in their environment and describe the geometric features:**
 - a. Identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone).
 - b. Compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners).

Statistics, Data Analysis, and Probability

7. Students collect information about objects and events in their environment:

- a. Pose information questions; collect data; and record the results using objects, pictures, and picture graphs.
- b. Identify, describe, and extend simple patterns (such as circles or triangles) by referring to their shapes, sizes, or colors.

Mathematical Reasoning

8. Students make decisions about how to set up a problem:

- a. Determine the approach, materials, and strategies to be used.
- b. Use tools and strategies, such as manipulatives or sketches, to model problems.

9. Students solve problems in reasonable ways and justify their reasoning:

- a. Explain the reasoning used with concrete objects and/ or pictorial representations.
- b. Make precise calculations and check the validity of the results in the context of the problem.

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Mathematics Content Standards

Grade One

By the end of grade one, students understand and use the concept of ones and tens in the place value number system. Students add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way.

Number Sense

- 1. Students understand and use numbers up to 100:**
 - a. Count, read, and write whole numbers to 100.
 - b. Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than ($<$, $=$, $>$).
 - c. Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 may be represented as $4 + 4$, $5 + 3$, $2 + 2 + 2 + 2$, $10 - 2$, $11 - 3$).
 - d. Count and group object in ones and tens (e.g., three groups of 10 and 4 equals 34, or $30 + 4$).
 - e. Identify and know the value of coins and show different combinations of coins that equal the same value.

- 2. Students demonstrate the meaning of addition and subtraction and use these operations to solve problems:**
 - a. Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.
 - b. Use the inverse relationship between addition and subtraction to solve problems.
 - c. Identify one more than, one less than, 10 more than, and 10 less than a given number.
 - d. Count by 2s, 5s, and 10s to 100.
 - e. Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).
 - f. Solve addition and subtraction problems with one-and two-digit numbers (e.g., $5 + 58 = \underline{\quad}$).
 - g. Find the sum of three one-digit numbers.

- 3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places:**
 - a. Make reasonable estimates when comparing larger or smaller numbers.

Algebra and Functions

- 4. Students use number sentences with operational symbols and expressions to solve problems:**
 - a. Write and solve number sentences from problem situations that express relationships involving addition and subtraction.
 - b. Understand the meaning of the symbols $+$, $-$, $=$.
 - c. Create problem situations that might lead to given number sentences involving addition and subtraction.

Measurement and Geometry

- 5. Students use direct comparison and nonstandard units to describe the measurements of objects:**

- a. Compare the length, weight, and volume of two or more objects by using direct comparison or a nonstandard unit.
- b. Tell time to the nearest half hour and relate time to events (e.g., before/after, shorter/longer).

6. Students identify common geometric figures, classify them by common attributes, and describe their relative position or their location in space:

- a. Identify, describe, and compare triangles, rectangles, squares, and circles, including the faces of three-dimensional objects.
- b. Classify familiar plane and solid objects by common attributes, such as color, position, shape, size, roundness, or number of corners, and explain which attributes are being used for classification.
- c. Give and follow directions about location.
- d. Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Statistics, Data Analysis, and Probability

7. Students organize, represent, and compare data by category on simple graphs and charts:

- a. Sort objects and data by common attributes and describe the categories.
- b. Represent and compare data (e.g., largest, smallest, most often, least often) by using pictures, bar graphs, tally charts, and picture graphs.

8. Students sort objects and create and describe patterns by numbers, shapes, sizes, rhythms, or colors:

- a. Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape).

Mathematical Reasoning

9. Students make decisions about how to set up a problem:

- a. Determine the approach, materials, and strategies to be used.
- b. Use tools, such as manipulatives or sketches, to model problems.

10. Students solve problems and justify their reasoning:

- a. Explain the reasoning used and justify the procedures selected.
- b. Make precise calculations and check the validity of the results from the context of the problem.

11. Students note connections between one problem and another.

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Mathematics Content Standards

Grade Two

By the end of grade two, students understand place value and number relationships in addition and subtraction, and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to their geometric attributes. They collect and analyze data and verify the answers. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way.

Number Sense

- 1. Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000:**
 - a. Count, read, and write whole numbers to 1,000 and identify the place value for each digit.
 - b. Use words, models, and expanded forms (e.g., $45 = 4 \text{ tens} + 5$) to represent numbers (to 1,000).
 - c. Order and compare whole numbers to 1,000 by using the symbols $<$, $=$, $>$.

- 2. Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers:**
 - a. Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for $8 + 6 = 14$ is $14 - 6 = 8$) to solve problems and check solutions.
 - b. Find the sum or difference of two whole numbers up to three digits long.
 - c. Use mental arithmetic to find the sum or difference of two two-digit numbers.

- 3. Students model and solve simple problems involving multiplication and division:**
 - a. Use repeated addition, arrays, and counting by multiples to do multiplication.
 - b. Use repeated subtraction, equal sharing, and forming equal groups with remainders to do division.
 - c. Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.

- 4. Students understand that fractions and decimals may refer to parts of a set and parts of a whole:**
 - a. Recognize, name, and compare unit fractions from $1/12$ to $1/2$.
 - b. Recognize fractions of a whole and parts of a group (e.g., one-fourth of a pie, two-thirds of 15 balls).
 - c. Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

- 5. Students model and solve problems by representing, adding, and subtracting amounts of money:**
 - a. Solve problems using combinations of coins and bills.
 - b. Know and use the decimal notation and the dollar and cent symbols for money.

- 6. Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds, and thousands places:**
 - a. Recognize when an estimate is reasonable in measurements (e.g., closest inch).

Algebra and Functions

- 7. Students model, represent, and interpret number relationships to create and solve problems involving addition and subtraction:**

- a. Use the commutative and associative rules to simplify mental calculations and to check results.
- b. Relate problem situations to number sentences involving addition and subtraction.
- c. Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.

Measurement and Geometry

8. Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit, and comparing it to the item to be measured:

- a. Measure the length of objects by iterating (repeating) a nonstandard or standard unit.
- b. Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.
- c. Measure the length of an object to the nearest inch and/ or centimeter.
- d. Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
- e. Determine the duration of intervals of time in hours (e.g., 11:00 a.m. to 4:00 p.m.).

9. Students identify and describe the attributes of common figures in the plane and of common objects in space:

- a. Describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges, and vertices.
- b. Put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).

Statistics, Data Analysis, and Probability

10. Students collect numerical data and record, organize, display, and interpret the data on bar graphs and other representations:

- a. Record numerical data in systematic ways, keeping track of what has been counted.
- b. Represent the same data set in more than one way (e.g., bar graphs and charts with tallies).
- c. Identify features of data sets (range and mode).
- d. Ask and answer simple questions related to data representations.

11. Students demonstrate an understanding of patterns and how patterns grow and describe them in general ways:

- a. Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on one horse, two horses, three horses, four horses).
- b. Solve problems involving simple number patterns.

Mathematical Reasoning

12. Students make decisions about how to set up a problem:

- a. Determine the approach, materials, and strategies to be used.
- b. Use tools, such as manipulatives or sketches, to model problems.

13. Students solve problems and justify their reasoning:

- a. Defend the reasoning used and justify the procedures selected.
- b. Make precise calculations and check the validity of the results in the context of the problem.

14. Students note connections between one problem and another.

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Mathematics Content Standards

Grade Three

By the end of grade three, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication, and division of whole numbers. Students estimate, measure, and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way.

Number Sense

- 1. Students understand the place value of whole numbers:**
 - a. Count, read, and write whole numbers to 10,000.
 - b. Compare and order whole numbers to 10,000.
 - c. Identify the place value for each digit in numbers to 10,000.
 - d. Round off numbers to 10,000 to the nearest ten, hundred, and thousand.
 - e. Use expanded notation to represent numbers (e.g., $3,206 = 3,000 + 200 + 6$).

- 2. Students calculate and solve problems involving addition, subtraction, multiplication, and division:**
 - a. Find the sum or difference of two whole numbers between 0 and 10,000.
 - b. Memorize to automaticity the multiplication table for numbers between 1 and 10.
 - c. Use the inverse relationship of multiplication and division to compute and check results.
 - d. Solve simple problems involving multiplication of multidigit numbers by one-digit numbers ($3,671 \times 3 = \underline{\quad}$).
 - e. Solve division problems in which a multidigit number is evenly divided by a one-digit number ($135 \div 5 = \underline{\quad}$).
 - f. Understand the special properties of 0 and 1 in multiplication and division.
 - g. Determine the unit cost when given the total cost and number of units.
 - h. Solve problems that require two or more of the skills mentioned above.

- 3. Students understand the relationship between whole numbers, simple fractions, and decimals:**
 - a. Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $1/2$ of a pizza is the same amount as $2/4$ of another pizza that is the same size; show that $3/8$ is larger than $1/4$).
 - b. Add and subtract simple fractions (e.g., determine that $1/8 + 3/8$ is the same as $1/2$).
 - c. Solve problems involving addition, subtraction, multiplication, and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.
 - d. Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $1/2$ of a dollar, 75 cents is $3/4$ of a dollar).

Algebra and Functions

- 4. Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships:**
 - a. Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities.
 - b. Solve problems involving numeric equations or inequalities.
 - c. Select appropriate operational and relational symbols to make an expression true (e.g., if $4 \underline{\quad} 3 = 12$, what operational symbol goes in the blank?).
 - d. Express simple unit conversions in symbolic form (e.g., $\underline{\quad}$ inches = $\underline{\quad}$ feet x 12).

- e. Recognize and use the commutative and associative properties of multiplication (e.g., if $5 \times 7 = 35$, then what is 7×5 ? and if $5 \times 7 \times 3 = 105$, then what is $7 \times 3 \times 5$?).

5. Students represent simple functional relationships:

- a. Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).
- b. Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).

Measurement and Geometry

6. Students choose and use appropriate units and measurement tools to quantify the properties of objects:

- a. Choose the appropriate tools and units (metric and U.S.) and estimate and measure the length, liquid volume, and weight/mass of given objects.
- b. Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.
- c. Find the perimeter of a polygon with integer sides.
- d. Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).

7. Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems:

- a. Identify, describe, and classify polygons (including pentagons, hexagons, and octagons).
- b. Identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle).
- c. Identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square).
- d. Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.
- e. Identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).
- f. Identify common solid objects that are the components needed to make a more complex solid object.

Statistics, Data Analysis, and Probability

8. Students conduct simple probability experiments by determining the number of possible outcomes and make simple predictions:

- a. Identify whether common events are certain, likely, unlikely, or improbable.
- b. Record the possible outcomes for a simple event (e.g., tossing a coin) and systematically keep track of the outcomes when the event is repeated many times.
- c. Summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot).
- d. Use the results of probability experiments to predict future events (e.g., use a line plot to predict the temperature forecast for the next day).

Mathematical Reasoning

9. Students make decisions about how to approach problems:

- a. Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
- b. Determine when and how to break a problem into simpler parts.

10. Students use strategies, skills, and concepts in finding solutions:

- a. Use estimation to verify the reasonableness of calculated results.

- b. Apply strategies and results from simpler problems to more complex problems.
- c. Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- d. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- e. Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- f. Make precise calculations and check the validity of the results from the context of the problem.

11. Students move beyond a particular problem by generalizing to other situations:

- a. Evaluate the reasonableness of the solution in the context of the original situation.
- b. Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- c. Develop generalizations of the results obtained and apply them in other circumstances.

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Mathematics Content Standards

Grade Four

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way. They also recognize how God allows us to use numbers to perform various calculations and appreciate how numbers work.

Number Sense

1. Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts of negative numbers:

- a. Read and write whole numbers in the millions.
- b. Order and compare whole numbers and decimals to two decimal places.
- c. Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.
- d. Decide when a rounded solution is called for and explain why such a solution may be appropriate.
- e. Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalence of fractions.
- f. Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., $1/2 = 0.5$ or $.50$; $7/4 = 1\ 3/4 = 1.75$).
- g. Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.
- h. Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").
- i. Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places.

2. Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals:

- a. Estimate and compute the sum or difference of whole numbers and positive decimals to two places.
- b. Round two-place decimals to one decimal or the nearest whole number and judge the reasonableness of the rounded answer.

3. Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:

- a. Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multi digit numbers.
- b. Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multi digit number by a two-digit number and for dividing a multi digit number by a one-digit number; use relationships between them to simplify computations and to check results.
- c. Solve problems involving multiplication of multi digit numbers by two-digit numbers.
- d. Solve problems involving division of multi digit numbers by one-digit numbers.

4. Students know how to factor small whole numbers:

- a. Understand that many whole numbers break down in different ways (e.g., $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$).

- b. Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.

Algebra and Functions

5. Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:

- a. Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).
- b. Interpret and evaluate mathematical expressions that now use parentheses.
- c. Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.
- d. Use and interpret formulas (e.g., area = length x width or $A = lw$) to answer questions about quantities and their relationships.
- e. Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

6. Students know how to manipulate equations:

- a. Know and understand that equals added to equals are equal.
- b. Know and understand that equals multiplied by equals are equal.

Measurement and Geometry

7. Students understand perimeter and area:

- a. Measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch (in^2), square yard (yd^2), or square mile (mi^2).
- b. Recognize that rectangles that have the same area can have different perimeters.
- c. Understand that rectangles that have the same perimeter can have different areas.
- d. Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

8. Students use two-dimensional coordinate grids to represent points and graph lines and simple figures:

- a. Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).
- b. Understand that the length of a horizontal line segment equals the difference of the x -coordinates.
- c. Understand that the length of a vertical line segment equals the difference of the y -coordinates.

9. Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems:

- a. Identify lines that are parallel and perpendicular.
- b. Identify the radius and diameter of a circle.
- c. Identify congruent figures.
- d. Identify figures that have bilateral and rotational symmetry.
- e. Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $3/4$, and full turns.
- f. Visualize, describe, and make models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

- g. Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes.
- h. Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).

Statistics, Data Analysis, and Probability

10. Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings:

- a. Formulate survey questions; systematically collect and represent data on a number line; and coordinate graphs, tables, and charts.
- b. Identify the mode(s) for sets of categorical data and the mode(s), median, and any apparent outliers for numerical data sets.
- c. Interpret one-and two-variable data graphs to answer questions about a situation.

11. Students make predictions for simple probability situations:

- a. Represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams).
- b. Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; $\frac{3}{4}$).

Mathematical Reasoning

12. Students make decisions about how to approach problems:

- a. Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
- b. Determine when and how to break a problem into simpler parts.

13. Students use strategies, skills, and concepts in finding solutions:

- a. Use estimation to verify the reasonableness of calculated results.
- b. Apply strategies and results from simpler problems to more complex problems.
- c. Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- d. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- e. Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- f. Make precise calculations and check the validity of the results from the context of the problem.

14. Students move beyond a particular problem by generalizing to other situations:

- a. Evaluate the reasonableness of the solution in the context of the original situation.
- b. Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- c. Develop generalizations of the results obtained and apply them in other circumstances.

St. Philip's Lutheran

Mathematics Content Standards

Grade Five

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions, decimals, and positive and negative numbers. They know and use common measuring units to determine length and area and know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way. They also recognize how God allows us to use numbers to perform various calculations and appreciate how numbers work.

Number Sense

- 1. Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions, and percents. They understand the relative magnitudes of numbers:**
 - a. Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
 - b. Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.
 - c. Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication.
 - d. Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).
 - e. Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

- 2. Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:**
 - a. Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
 - b. Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
 - c. Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
 - d. Understand the concept of multiplication and division of fractions.
 - e. Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Algebra and Functions

- 3. Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:**
 - a. Use information taken from a graph or equation to answer questions about a problem situation.
 - b. Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.
 - c. Know and use the distributive property in equations and expressions with variables.
 - d. Identify and graph ordered pairs in the four quadrants of the coordinate plane.

- e. Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

Measurement and Geometry

4. Students understand and compute the volumes and areas of simple objects:

- a. Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by cutting and pasting a right triangle on the parallelogram).
- b. Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.
- c. Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [in^3], cubic yard [yd^3]) to compute the volume of rectangular solids.
- d. Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

5. Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

- a. Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).
- b. Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.
- c. Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Statistics, Data Analysis, and Probability

6. Students display, analyze, compare, and interpret different data sets, including data sets of different sizes:

- a. Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.
- b. Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.
- c. Use fractions and percentages to compare data sets of different sizes.
- d. Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.
- e. Know how to write ordered pairs correctly; for example, (x, y) .

Mathematical Reasoning

7. Students make decisions about how to approach problems:

- a. Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
- b. Determine when and how to break a problem into simpler parts.

8. Students use strategies, skills, and concepts in finding solutions:

- a. Use estimation to verify the reasonableness of calculated results.
- b. Apply strategies and results from simpler problems to more complex problems.
- c. Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- d. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

- e. Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- f. Make precise calculations and check the validity of the results from the context of the problem.

9. Students move beyond a particular problem by generalizing to other situations:

- a. Evaluate the reasonableness of the solution in the context of the original situation.
- b. Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- c. Develop generalizations of the results obtained and apply them in other circumstances.

St. Philip's Lutheran

Mathematics Content Standards

Grade Six

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about pi and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way. They also recognize how God allows us to use numbers to perform various calculations and appreciate how numbers work.

Number Sense

- 1. Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:**
 - a. Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.
 - b. Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b , a to b , $a:b$).
 - c. Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.
 - d. Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

- 2. Students calculate and solve problems involving addition, subtraction, multiplication, and division:**
 - a. Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
 - b. Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $5/8 \div 15/16 = 5/8 \times 16/15 = 2/3$).
 - c. Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.
 - d. Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Algebra and Functions

- 3. Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:**
 - a. Write and solve one-step linear equations in one variable.
 - b. Write and evaluate an algebraic expression for a given situation, using up to three variables.
 - c. Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

- d. Solve problems manually by using the correct order of operations or by using a scientific calculator.
- 4. Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:**
 - a. Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).
 - b. Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.
 - c. Solve problems involving rates, average speed, distance, and time.
 - 5. Students investigate geometric patterns and describe them algebraically:**
 - a. Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = 1/2bh$, $C = \pi d$ - the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).
 - b. Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry

- 6. Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:**
 - a. Understand the concept of a constant such as π ; know the formulas for the circumference and area of a circle.
 - b. Know common estimates of π (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.
 - c. Know and use the formulas for the volume of triangular prisms and cylinders (area of base \times height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.
- 7. Students identify and describe the properties of two-dimensional figures:**
 - a. Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.
 - b. Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.
 - c. Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

- 8. Students compute and analyze statistical measurements for data sets:**
 - a. Compute the range, mean, median, and mode of data sets.
 - b. Understand how additional data added to data sets may affect these computations of measures of central tendency.
 - c. Understand how the inclusion or exclusion of outliers affects measures of central tendency.
 - d. Know why a specific measure of central tendency (mean, median) provides the most useful information in a given context.
- 9. Students use data samples of a population and describe the characteristics and limitations of the samples:**
 - a. Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.
 - b. Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

- c. Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.
- d. Identify data that represent sampling errors and explain why the sample (and the display) might be biased.
- e. Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

10. Students determine theoretical and experimental probabilities and use these to make predictions about events:

- a. Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.
- b. Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).
- c. Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1 - P$ is the probability of an event not occurring.
- d. Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.
- e. Understand the difference between independent and dependent events.

Mathematical Reasoning

11. Students make decisions about how to approach problems:

- a. Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
- b. Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
- c. Determine when and how to break a problem into simpler parts.

12. Students use strategies, skills, and concepts in finding solutions:

- a. Use estimation to verify the reasonableness of calculated results.
- b. Apply strategies and results from simpler problems to more complex problems.
- c. Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
- d. Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- e. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- f. Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- g. Make precise calculations and check the validity of the results from the context of the problem.

13. Students move beyond a particular problem by generalizing to other situations:

- a. Evaluate the reasonableness of the solution in the context of the original situation.
- b. Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

- c. Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

St. Philip's Lutheran

Mathematics Content Standards

Grade Seven

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way. They also recognize how God allows us to use numbers to perform various calculations and appreciate how numbers work.

Number Sense

1. Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

- a. Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.
- b. Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- c. Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- d. Differentiate between rational and irrational numbers.
- e. Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.
- f. Calculate the percentage of increases and decreases of a quantity.
- g. Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

2. Students use exponents, powers, and roots and use exponents in working with fractions:

- a. Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.
- b. Add and subtract fractions by using factoring to find common denominators.
- c. Multiply, divide, and simplify rational numbers by using exponent rules.
- d. Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.
- e. Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions

3. Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

- a. Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
- b. Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

- c. Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.
- d. Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.
- e. Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

4. Students interpret and evaluate expressions involving integer powers and simple roots:

- a. Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.
- b. Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

5. Students graph and interpret linear and some nonlinear functions:

- a. Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.
- b. Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).
- c. Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.
- d. Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

6. Students solve simple linear equations and inequalities over the rational numbers:

- a. Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- b. Solve multi step problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry

7. Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

- a. Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).
- b. Construct and read drawings and models made to scale.
- c. Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

8. Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

- a. Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.
- b. Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.
- c. Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all

dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

- d. Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units ($1 \text{ square foot} = 144 \text{ square inches}$ or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately $16.38 \text{ cubic centimeters}$ or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$).

9. Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

- a. Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.
- b. Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.
- c. Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.
- d. Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.
- e. Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.
- f. Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Statistics, Data Analysis, and Probability

10. Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

- a. Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.
- b. Represent two numerical variables on a scatter plot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).
- c. Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Mathematical Reasoning

11. Students make decisions about how to approach problems:

- a. Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
- b. Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
- c. Determine when and how to break a problem into simpler parts.

12. Students use strategies, skills, and concepts in finding solutions:

- a. Use estimation to verify the reasonableness of calculated results.
- b. Apply strategies and results from simpler problems to more complex problems.
- c. Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
- d. Make and test conjectures by using both inductive and deductive reasoning.

- e. Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- f. Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- g. Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- h. Make precise calculations and check the validity of the results from the context of the problem.

13. Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:

- a. Evaluate the reasonableness of the solution in the context of the original situation.
- b. Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- c. Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

St. Philip's Lutheran

Mathematics Content Standards

Grade Eight

By grade eight, students' mathematical sensitivity should be sharpened. Students need to start perceiving logical subtleties and appreciate the need for sound mathematical arguments before making conclusions. As students progress in the study of mathematics, they learn to distinguish between inductive and deductive reasoning; understand the meaning of logical implication; test general assertions; realize that one counterexample is enough to show that a general assertion is false; understand conceptually that although a general assertion is true in a few cases, it is not true in all cases; distinguish between something being proven and a mere plausibility argument; and identify logical errors in chains of reasoning.

Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline students master at more advanced levels. Students use their God-given, logical minds to examine these things and to develop an appreciation for how God created our world in such an orderly way. They also recognize how God allows us to use numbers to perform various calculations and appreciate how numbers work.

Algebra

- 1. Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:**
 - a. Students use properties of numbers to demonstrate whether assertions are true or false.
- 2. Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.**
- 3. Students solve equations and inequalities involving absolute values.**
- 4. Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.**
- 5. Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.**
- 6. Students graph a linear equation and compute the x - and y -intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).**
- 7. Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.**
- 8. Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.**
- 9. Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.**
- 10. Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.**
- 11. Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.**
- 12. Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.**
- 13. Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.**

14. Students solve a quadratic equation by factoring or completing the square.
15. Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
16. Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17. Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18. Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
19. Students know the quadratic formula and are familiar with its proof by completing the square.
20. Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
21. Students graph quadratic functions and know that their roots are the x -intercepts.
22. Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x -axis in zero, one, or two points.
23. Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
24. Students use and know simple aspects of a logical argument:
 - a. Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
 - b. Students identify the hypothesis and conclusion in logical deduction.
 - c. Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.
25. Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:
 - a. Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
 - b. Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
 - c. Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.
26. Students solve equations and inequalities involving absolute value.
27. Students solve systems of linear equations and inequalities (in two or three variables) by substitution, or with graphs.
28. Students are adept at operations on polynomials, including long division.
29. Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.
30. Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.
31. Students add, subtract, multiply, and divide complex numbers.
32. Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.
33. Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.
34. Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.
35. Students use combinations and permutations to compute probabilities.
36. Students apply the method of mathematical induction to prove general statements about the positive integers.
37. Students use properties from number systems to justify steps in combining and simplifying functions.

Probability and Statistics

- 38. Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.**
- 39. Students demonstrate an understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.**
- 40. Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.**
- 41. Students determine the mean and the standard deviation of a normally distributed random variable.**
- 42. Students know the definitions of the mean, median, and mode of a distribution of data and can compute each in particular situations.**
- 43. Students compute the variance and the standard deviation of a distribution of data.**
- 44. Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.**

Glossary

absolute value. A number's distance from zero on the number line. The absolute value of -4 is 4; the absolute value of 4 is 4.

algorithm. An organized procedure for performing a given type of calculation or solving a given type of problem. An example is long division.

arithmetic sequence. A sequence of elements, a_1, a_2, a_3, \dots , such that the difference of successive terms is a constant $a_{i+1} - a_i = k$; for example, the sequence $\{2, 5, 8, 11, 14, \dots\}$ where the common difference is 3.

asymptotes. Straight lines that have the property of becoming and staying arbitrarily close to the curve as the distance from the origin increases to infinity. For example, the x -axis is the only asymptote to the graph of $\sin(x)/x$.

axiom. A basic assumption about a mathematical system from which theorems can be deduced. For example, the system could be the points and lines in the plane. Then an axiom would be that given any two distinct points in the plane, there is a unique line through them.

binomial. In algebra, an expression consisting of the sum or difference of two monomials (see the definition of *monomial*), such as $4a-8b$.

binomial distribution. In probability, a binomial distribution gives the probabilities of k outcomes A (or $n-k$ outcomes B) in n independent trials for a two-outcome experiment in which the possible outcomes are denoted A and B .

binomial theorem. In mathematics, a theorem that specifies the complete expansion of a binomial raised to any positive integer power.

box-and-whisker plot. A graphical method for showing the median, quartiles, and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

complex numbers. Numbers that have the form $a + bi$ where a and b are real numbers and i satisfies the equation $i^2 = -1$. Multiplication is denoted by $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$, and addition is denoted by $(a + bi) + (c + di) = (a + c) + (b + d)i$.

congruent. Two shapes in the plane or in space are congruent if there is a rigid motion that identifies one with the other (see the definition of *rigid motion*).

conjecture. An educated guess.

coordinate system. A rule of correspondence by which two or more quantities locate points unambiguously and which satisfies the further property that points unambiguously determine the quantities; for example, the usual Cartesian coordinates x, y in the plane.

cosine. $\cos(q)$ is the x -coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of q with the positive x -axis. When q is an angle of a right triangle, then $\cos(q)$ is the ratio of the adjacent side with the hypotenuse.

dilation. In geometry, a transformation D of the plane or space is a dilation at a point P if it takes P to itself, preserves angles, multiplies distances from P by a positive real number r , and takes

every ray through P onto itself. In case P is the origin for a Cartesian coordinate system in the plane, then the dilation D maps the point (x, y) to the point (rx, ry) .

dimensional analysis. A method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. For example, velocity has units of the form length over time (e.g., meters per second [m/sec]), and acceleration has units of velocity over time; so it follows that acceleration has units $(m/sec)/sec = m/(sec^2)$.

expanded form. The expanded form of an algebraic expression is the *equivalent expression* without parentheses. For example, the expanded form of $(a + b)^2$ is $a^2 + 2ab + b^2$.

exponent. The power to which a number or variable is raised (the exponent may be any real number).

exponential function. A function commonly used to study growth and decay. It has the form $y = a^x$ with a positive.

factors. Any of two or more quantities that are multiplied together. In the expression 3.712×11.315 , the factors are 3.712 and 11.315.

function. A correspondence in which values of one variable determine the values of another.

geometric sequence. A sequence in which there is a common ratio between successive terms. Each successive term of a geometric sequence is found by multiplying the preceding term by the common ratio. For example, in the sequence $\{1, 3, 9, 27, 81, \dots\}$ the common ratio is 3.

histogram. A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

inequality. A relationship between two quantities indicating that one is strictly *less than* or *less than or equal* to the other.

integers. The set consisting of the positive and negative whole numbers and zero; for example, $\{\dots, -2, -1, 0, 1, 2, \dots\}$.

irrational number. A number that cannot be represented as an exact ratio of two integers. For example, the square root of 2 or p .

linear expression. An expression of the form $ax+b$ where x is variable and a and b are constants; or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$, etc.

linear equation. An equation containing linear expressions.

logarithm. The inverse of exponentiation; for example, $a^{\log_a x} = x$.

mean. In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

median. In statistics, the quantity designating the middle value in a set of numbers.

mode. In statistics, the value that occurs most frequently in a given series of numbers.

monomial. In the variables x, y, z , a monomial is an expression of the form $ax^m y^n z^k$, in which m, n , and k are nonnegative integers and a is a constant (e.g., $5x^2$, $3x^2 y$ or $7x^3 yz^2$).

nonstandard unit. Unit of measurement expressed in terms of objects (such as paper clips, sticks of gum, shoes, etc.).

parallel. Given distinct lines in the plane that are infinite in both directions, the lines are parallel if they never meet. Two distinct lines in the coordinate plane are parallel if and only if they have the same slope.

permutation. A permutation of the set of numbers $\{1, 2, \dots, n\}$ is a reordering of these numbers.

polar coordinates. The coordinate system for the plane based on r, θ , the distance from the origin and θ , and the angle between the positive x -axis and the ray from the origin to the point.

polar equation. Any relation between the polar coordinates (r, θ) of a set of points (e.g., $r = 2\cos\theta$ is the polar equation of a circle).

polynomial. In algebra, a sum of monomials; for example, $x^2 + 2xy + y^2$.

prime. A natural number p greater than 1 is prime if and only if the only positive integer factors of p are 1 and p . The first seven primes are 2, 3, 5, 7, 11, 13, 17.

quadratic function. A function given by a polynomial of degree 2.

random variable. A function on a probability space.

range. In statistics, the difference between the greatest and smallest values in a data set. In mathematics, the image of a function.

ratio. A comparison expressed as a fraction. For example, there is a ratio of three boys to two girls in a class ($3/2$, $3:2$).

rational numbers. Numbers that can be expressed as the quotient of two integers; for example, $7/3$, $5/11$, $-5/13$, $7 = 7/1$.

real numbers. All rational and irrational numbers.

reflection. The reflection through a line in the plane or a plane in space is the transformation that takes each point in the plane to its mirror image with respect to the line or its mirror image with respect to the plane in space. It produces a mirror image of a geometric figure.

rigid motion. A transformation of the plane or space, which preserves distance and angles.

root extraction. Finding a number that can be used as a factor a given number of times to produce the original number; for example, the fifth root of 32 = 2 because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

rotation. A rotation in the plane through an angle θ and about a point P is a rigid motion T fixing P so that if Q is distinct from P , then the angle between the lines PQ and $PT(Q)$ is always θ . A rotation through an angle θ in space is a rigid motion T fixing the points of a line l so that it is a rotation through θ in the plane perpendicular to l through some point on l .

scalar matrix. A matrix whose diagonal elements are all equal while the non diagonal elements are all 0. The identity matrix is an example.

scatterplot. A graph of the points representing a collection of data.

scientific notation. A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (e.g., $7000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).

similarity. In geometry, two shapes R and S are similar if there is a dilation D (see the definition of *dilation*) that takes S to a shape congruent to R . It follows that R and S are similar if they are congruent after one of them is expanded or shrunk.

sine. $\sin(q)$ is the y -coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of q with the positive x -axis. When q is an angle of a right triangle, then $\sin(q)$ is the ratio of the opposite side with the hypotenuse.

square root. The square roots of n are all the numbers m so that $m^2 = n$. The square roots of 16 are 4 and -4. The square roots of -16 are $4i$ and $-4i$.

standard deviation. A statistic that measures the dispersion of a sample.

symmetry. A symmetry of a shape S in the plane or space is a rigid motion T that takes S onto itself ($T(S) = S$). For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

system of linear equations. Set of equations of the first degree (e.g., $x + y = 7$ and $x - y = 1$). A solution of a set of linear equations is a set of numbers a, b, c, \dots so that when the variables are replaced by the numbers all the equations are satisfied. For example, in the equations above, $x = 4$ and $y = 3$ is a solution.

translation. A rigid motion of the plane or space of the form X goes to $X + V$ for a fixed vector V .

transversal. In geometry, given two or more lines in the plane a transversal is a line distinct from the original lines and intersects each of the given lines in a single point.

unit fraction. A fraction whose numerator is 1 (e.g., $1/p$, $1/3$, $1/x$). Every nonzero number may be written as a unit fraction since, for n not equal to 0, $n = 1/(1/n)$.

variable. A placeholder in algebraic expressions; for example, in $3x + y = 23$, x and y are variables.

vector. Quantity that has magnitude (length) and direction. It may be represented as a directed line segment.

zeros of a function. The points at which the value of a function is zero.